Simulated Annealing Techniques for Joint Transmitter-Receiver Design in a Multiple User Access MIMO-OFDM Channel

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ABSTRACT

In this paper we consider the problem of the joint design of multiple transmitters and receivers in a MIMO-OFDM scenario. We consider that both the transmitters and receivers have multiple antennas, configuring a Multi-Input-Multi-Output (MIMO) channel. Based on this structure, space diversity techniques can be used to separate the signals transmitted by different users. Concretely, the proposed approach consists in beamforming at both sides of the system, combined with the Orthogonal-Frequency-Division-Multiplexing (OFDM) modulation. The goal is the minimization of the total transmitted power subject to constraints related to the Quality of Service (QoS) for each user in terms of the Bit Error Rate (BER) and the maximum transmitted power for some terminals. By making use of this approach, the Multiple Access Interference (MAI) is minimized and the physical resources are exploited in an optimum way. The design is based on Simulated Annealing (SA), an heuristic technique able to find the optimum design in this non-convex problem and that can take into account all kind of constraints.

I. INTRODUCTION

In last years mobile communications have experimented an exponential growth in terms of capacity requirements or number of users, bit-rate increase and Quality of Service (QoS) or Bit Error Rate (BER) improvement. Space diversity based structures represent a good collection of techniques to cope with these challenges. Concretely, the Multi-Input-Multi-Output (MIMO) channels, based on the utilization of multiple antennas at both the transmitters and receivers, have been the target of study of many researchers. Moreover, many new standards such as the European Wireless Local Area Network (WLAN) HIPERLAN/2 [1] have proposed to use the Orthogonal-Frequency-Division-Multiplexing (OFDM) as their modulation format. Many works have considered the combination of MIMO and OFDM from a single-user point of view [2] [3], but this structure is also able to implement the Space

Division Multiple Access (SDMA), in which several users accede to the radio channel at the same time and at the same frequency, providing a very high spectral efficiency able to accommodate important increases in system capacity.

In this work we propose a multiple user system, in which several Mobile Terminals (MTs) with more than one antenna are coexisting in the same area. These users are assumed to use the same frequency and accede to the channel at the same time by means of the OFDM modulation. The receiver equipments, for example the Base Stations (BSs) in an uplink transmission, are also assumed to have more than one antenna and must detect the transmitted symbols by all the users. The approach taken into account in this work for designing the transmitters and receivers is joint beamforming. For example, in an uplink communication with one BS, the transmitters use a beamformer for each frequency or carrier, and the receiver has a bank of beamformers for each carrier in order to detect the transmitted symbols by all the users. Because of simplicity and computational load reasons, we consider that linear single-user detectors are implemented at the receiver side.

The stated problem consists in the design of the beamformers subject to different constraints related to the Quality of Service (QoS) for each communication in terms of the Bit Error Rate (BER). Our goal is to minimize the total transmitted power so as to guarantee an optimum utilization of the physical resources. As in the uplink channel some of the MTs may have a limited battery capacity, we also permit to add constraints corresponding to the maximum transmit power for some of the MTs. This optimization problem is not easy to solve and closed expressions do not exist. Besides, it is not convex, and so classical approaches based on gradient algorithms may find a suboptimal design. In this work we propose the application of Simulated Annealing (SA) to solve this optimization problem. SA is an heuristic technique able to find the global optimum subject to all kind of constraints.

This paper is organized as follows. In Section II the system and signal models are presented. Section III

applies SA to the stated design problem, whereas in Section IV an alternative Lagrange-gradient based algorithm is proposed. Finally, in Section V some simulation results and conclusions are obtained.

II. SYSTEM AND SIGNAL MODELS

A. System and Signal Models

We consider a wireless scenario with several users acceding to the radio channel at the same time and the same frequency. A typical example corresponds to an uplink transmission in a cellular system such as that shown in Fig. 1.
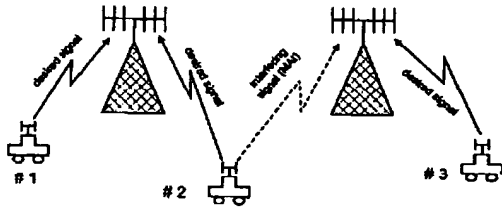

Fig. 1. Typical configuration in a multiuser MIMO-OFDM scenario with 3 users or communications.

The most general model would correspond to an ad-hoc network, in which all the devices can behave as a transmitter and receiver simultaneously, and each communication or link is engaged to a different transmitter and receiver, including possible multihop routing. In this paper we concentrate on the case of an uplink transmission in which K MTs are transmitting simultaneously in time and frequency to the same BS, that is responsible for detecting the information symbols transmitted by all the K users. As it was stated before, we assume that both the MTs and BS are equipped with more than one antenna. The extension to other kinds of scenarios or networks is direct and simple [4].

The snapshot vector signal model $y_n(t)$ at the nth carrier or frequency at the BS is [2]:

$$y_n(t) = \sum_{l=1}^{K} H_n^{(l)} b_n^{(l)} s_n^{(l)}(t) + n_n(t) \qquad (1)$$

where the number of elements of $y_n(t)$ is equal to the number of antennas of the BS. $b_n^{(l)}$ is the transmit beamvector corresponding to the lth MT and nth carrier, and has as many components as number of antennas in the associated MT. $s_n^{(l)}(t)$ is the transmitted symbol by the lth user at the nth carrier during the tth OFDM symbol period. It is assumed that the information data are normalized: $E\left\{\left|s_n^{(l)}(t)\right|^2\right\} = 1$, where $E\{\cdot\}$ stands for the mathematical expectation operator. The matrix $H_n^{(l)}$ represents the MIMO channel between the lth MT and the BS at the nth carrier [2]. Finally, the snapshot vector $n_n(t)$ models the contribution of noise plus interferences from outside the system at the BS at the same frequency. The associated covariance matrix is:

$$\varphi_n = E\left\{n_n(t) n_n^H(t)\right\}, \quad \text{where} \quad (\cdot)^H \quad \text{stands for}$$
conjugate transpose.

B. Single-User Receiver Optimization

The BS receives the signals transmitted by all the MTs. Its goal is to detect or estimate the information symbols transmitted by all the users. Naturally, the optimum receiver should be based on a multiuser detector; however, as its computational load is too high and possibly prohibitive, in this work we propose the utilization of a bank of linear single-user receivers, where each of them considers the other users as interferences. The optimum receive beamvector $a_n^{(k)}$ for the kth user at the nth frequency is the one that maximizes the Signal-to-Noise and Interference Ratio (SNIR), and corresponds to the well-known Matched Filter (MF) [2]:

$$a_n^{(k)} = \alpha_n^{(k)} R_n^{(k)^{-1}} H_n^{(k)} b_n^{(k)} \qquad (2)$$

$$R_n^{(k)} = \varphi_n + \sum_{\substack{l=1 \\ l \neq k}}^{K} H_n^{(l)} b_n^{(l)} b_n^{(l)^H} H_n^{(l)^H} \qquad (3)$$

where $\alpha_n^{(k)}$ is a constant that does not affect the SNIR and that can be calculated so as to have well adjusted decision threshold levels.

The estimates of the transmitted symbols are calculated as follows: $\hat{s}_n^{(k)}(t) = dec\left\{a_n^{(k)^H} y_n(t)\right\}$, where $dec\{\cdot\}$ stands for decision. Taking into account this design, the SNIR at the output of the beamformer at the nth carrier for the kth user can be expressed as:

$$SNIR_n^{(k)} = b_n^{(k)^H} H_n^{(k)^H} R_n^{(k)^{-1}} H_n^{(k)} b_n^{(k)} \qquad (4)$$

In OFDM it is defined the effective $BER^{(k)}$ for the kth user as the BER averaged over the subcarriers (here we assume that N carries are active for each user):

$$BER^{(k)} = \frac{1}{N} \sum_{n=0}^{N-1} Q\left(\sqrt{k_m SNIR_n^{(k)}}\right) \qquad (5)$$

where k_m is a constant depending on the modulation format for each subcarrier (for BPSK, $k_m = 2$) and we have assumed that the interferences are approximately Gaussian distributed.

III. SIMULATED ANNEALING BASED TRANSMITTER OPTIMIZATION

In last section we derived the optimum linear single-user receivers. In an uplink scenario, the BS has a bank of K single-user receivers, each of them responsible for detecting the symbols of one user, while considering the others as interferences. In this section our goal is to design the transmit beamvectors $b_n^{(k)}$ subject to QoS constraints. In this paper these QoS are directly expressed in terms of the target effective BER. Obviously, an important parameter in a wireless scenario is the transmitted power, specially in an uplink transmission or an ad-hoc network, in which the

transmitters are MTs with a limited battery capacity. Because of this, we also take into account these considerations in the design.

The power used for transmitting the signal at the nth frequency by the kth user is proportional to $\left\|b_n^{(k)}\right\|^2$.

The optimization problem consists in minimizing the total transmitted power P_T (6) subject to the QoS constraints, which indicate that the effective BER for each user must be lower than or equal to a target prefixed value $\gamma^{(k)}$ (7), guaranteeing a minimum QoS:

$$P_T = \sum_{k=1}^{K} \sum_{n=0}^{N-1} \left\|b_n^{(k)}\right\|^2 \qquad (6)$$

$$BER^{(k)} \le \gamma^{(k)} \qquad k=1,...,K \qquad (7)$$

In addition to the QoS constraints (7) and as explained before, it could be convenient to add constraints corresponding to the maximum transmit powers for some of the MTs $P_{max}^{(i)}$, specially in the uplink channel and when the battery capacity is very limited. Let us define Υ as the set of MTs to which the maximum transmit power constraints are applied. This constraints are expressed as shown in (8):

$$P_T^{(i)} = \sum_{n=0}^{N-1} \left\|b_n^{(i)}\right\|^2$$

$$P_T^{(i)} \le P_{max}^{(i)} \qquad i \in \Upsilon \qquad (8)$$

Closed expressions do not exist for this constrained optimization problem, and so, it must be solved by means of iterative methods.

This problem formulation generalizes that presented in [5]. There, the design was based on a multiuser Multicarrier CDMA (MC-CDMA) system with only one antenna at both the transmitter and receiver, and the QoS constraints were applied directly in terms of the SNIR. The problem was solved by means of a gradient-based algorithm. In [6] the transmitted power, which was a prefixed value, was the same for all the users and carriers; therefore no power allocation was carried out. The goal was the optimization of the SNIR by means of an iterative algorithm (Alternate & Maximize - AM). [7] generalized the iterative technique presented in [6] to more general conditions and assumptions. The main problem of [5], [6] and [7] is that a non-optimum solution, that is, a local minimum may be achieved due to the non-convexity behavior of this optimization problem [7]. Besides, neither of them considered the individual transmit powers constraints (8). The gradient technique presented in [5] was based on the Lagrange multipliers method and the quadratic penalty function [8]. This technique may have some convergence problems related to the speed of convergence and the convergence itself [5] [8]. Besides, the gradient requires that the constraints are differentiable, limiting its application. In the simulation results section, we compare a gradient-based technique with the proposed algorithm in this paper, based on Simulated Annealing (SA).

In this paper we propose the application of SA to solve the stated optimization problem taking into account all the constraints previously presented. SA is an iterative algorithm able to find the global optimum solution, even when the problem is non-convex, provided a feasible solution exists, that is, a solution that satisfies all the constraints simultaneously. If a feasible solution does not exist, then the algorithm will not converge to any acceptable design. SA has analogies with the annealing in physics as explained in [9]. In our problem, in each step there is a collection of beamvectors $\left\{b_n^{(k)}\right\}_{n=0,...,N-1}^{k=1,...,K}$ which is called the current solution. Given the current solution (equivalent to a concrete particles arrangement or state in physics), a new solution or collection of beamvectors is proposed. If it is "better" than the previous one, then it is accepted as the current solution. On the contrary, if it is "worse", then the proposed solution is accepted with a certain probability. This mechanism, called "hill-climbing", is important as it avoids finding a suboptimal solution or local minimum. The parameter that controls this acceptance probability is the temperature T, as in the case of annealing in physics. The higher the temperature, the higher the acceptance probability. The temperature T is lowered step by step, so that asymptotically only "better" solutions are accepted and a minimum is achieved. The meaning of "better" and "worse" is related to the definition of a cost function $f(\cdot)$ that depends on the transmit beamvectors (9), and which corresponds to the energy of a state in the annealing in physics.

Here we summarize the basic ideas of the SA algorithm that we propose to solve the stated optimization problem. In this case, we apply the algorithm to a continuous solution space. Besides, the cost function $f(\cdot)$ depends on the temperature T (9):

- *Cost function definition:*

$$f\left(\left\{b_n^{(k)}\right\}\right) = P_T + \frac{\alpha}{T} \sum_{k=1}^{K} \left(\log \frac{BER^{(k)}}{\gamma^{(k)}} \right)^{+2} + \frac{\alpha}{T} \sum_{i \in \Upsilon} \left(\log \frac{P_T^{(i)}}{P_{max}^{(i)}} \right)^{+2} \qquad (9)$$

where $(x)^+ = \max\{0,x\}$.

- *Proposed solution generation:*

$$\hat{b}_n^{(k)} = b_n^{(k)} + w_n^{(k)} \qquad n=0,...,N-1, k=1,...,K$$

$$w_n^{(k)} \sim CN\left(0, \sigma_b^2 I\right) \qquad (10)$$

- *Probability of acceptance of the proposed solution (Metropolis criterion)* [9]:

$$Pr = \exp\left\{ -\frac{\left(f\left(\left\{\hat{b}_n^{(k)}\right\}\right) - f\left(\left\{b_n^{(k)}\right\}\right)\right)^+}{T} \right\} \qquad (11)$$

- *System cooling:* the temperature is lowered with an exponential profile:

$$T \leftarrow \beta T \qquad \beta = 0.99 \qquad (12)$$

Initially the temperature T must be high enough so that most of the proposed solutions are accepted (in our case, we increase the temperature until the acceptance ratio is 95%). In this work we run 100 iterations per each value of T. The cost function is equal to the total transmit power plus a quadratic penalty term that takes into account if the effective BERs are greater than the required ones, and if the individual transmit powers are greater than those specified. As T is lowered, the penalty term is increased, and so, we asymptotically avoid solutions that do not fulfill the constraints. We make relative comparisons of the effective BER and transmit powers with the required values by means of the log(·) function, as experimentally we have observed that it behaves better than absolute comparisons. Nevertheless, other kind of comparison functions could have been used. The proposed solutions are generated by applying complex circularly symmetric Gaussian noise to the components of the beamvectors. The acceptance ratio is checked per each value of T. In case it is lower than 0.1 for 5 times, then the variance of the Gaussian noise is lowered by means of an exponential rule $\left(\sigma_b^2 \leftarrow 0.95\sigma_b^2\right)$. This helps the algorithm to find with more precision the minimum as the temperature is lowered, and therefore, it increases significantly the convergence rate of the technique.

IV. LAGRANGE-GRADIENT BASED TRANSMITTER OPTIMIZATION

In this section an alternative solution based on the Lagrange multiplier method and the quadratic penalty function [5] [8] is presented. For simplicity, here we only take into account the constraints corresponding to the BER. It can be easily shown that the minimum transmit power can be achieved when these constraints are fulfilled with equality, and so, we apply this result directly. The gradient algorithm is based on the definition of the following Lagrangian expression L:

$$L = P_T + \lambda \sum_{k=1}^{K} \left(\log \frac{BER^{(k)}}{\gamma^{(k)}} \right)^2 \qquad (13)$$

An iterative process is applied so as to find the optimum weight vectors $b_n^{(k)}$. The penalty parameter λ is calculated by means of a gradient ascent technique, whereas the beamvectors are calculated with a descent gradient approach. In both cases the parameter step-size μ must be used:

$$\lambda \leftarrow \lambda + \mu \sum_{k=1}^{K} \left(\log \frac{BER^{(k)}}{\gamma^{(k)}} \right)^2 \qquad (14)$$

$$b_n^{(k)} \leftarrow b_n^{(k)} - \mu \nabla_{b_n^{(k)}} L \qquad (15)$$

For space reasons, the expressions corresponding to $\nabla_{b_n^{(k)}} L$ are not shown here, although they can be easily obtained by using the results deduced in [5].

A problem of this technique is that no "a-priori" value of μ can be obtained for assuring the convergence in a reasonable number of iterations, and that a local minimum may be achieved instead of the global one. Besides, as it is explained in [5] and [8], the speed of this gradient-based technique decreases importantly as the design approaches a solution fulfilling the constraints, and therefore, in a reasonable time, it cannot be assured that it is possible to find a design fulfilling the constraints with an allowable error.

V. SIMULATION RESULTS AND CONCLUSIONS

In this section we simulate an uplink channel with 3 MTs and 1 BS. The OFDM modulation consists of 16 carriers and both the BS and MTs have 5 antennas. The QoS constraints in terms of the mean uncoded effective BER are: 10^{-3}, 10^{-3} and 10^{-2} for each user, and $\alpha=100$.

For all the figures presented in this section we show the behavior of the techniques versus the number of "flops" in MATLAB, so that we can compare the algorithms directly on a computational load basis.

In the first scenario we assume that the path loss is very similar for all the users. In Fig. 2 we show the power for the 3 users and the BER when no constraints on the individual transmit powers are applied. It is concluded that the SA algorithm is able to find a design fulfilling the QoS requirements. The power allocated to the first user is 8.45 W, and the total power is 20.1 W.
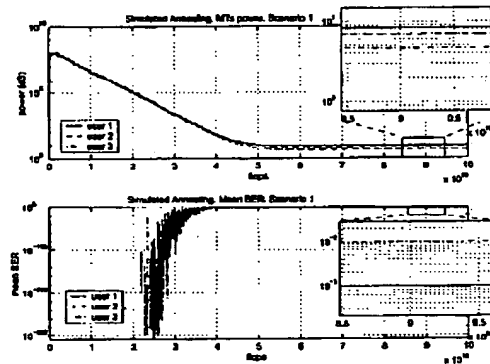

Fig. 2. Performance of the Simulated Annealing algorithm in scenario 1.

As it can be seen in the figure above, during the first iterations the transmit powers allocated to the MTs are very high, and as a direct consequence, the effective BERs are extremely low. This solution is not efficient, as we can lower the transmit powers by increasing the BERs until the QoS constraints are fulfilled with equality. This behavior is shown in the figure.

If we apply a power constraint to the first user equal to 8 W, then the results obtained by SA are those shown in Fig. 3. We conclude that in this case the algorithm allocates 7.6 W to the first user, whereas the others increase their corresponding power consumption. The global transmit power has increased up to 20.8 W.
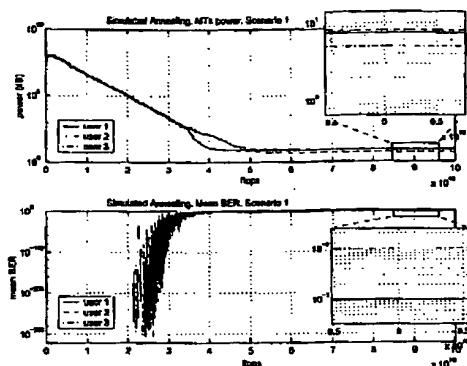
Fig. 3. Performance of the Simulated Annealing
algorithm in scenario 1. Power constraint on MT 1.

In Fig. 4 and 5 we show the results for an scenario
in which the third user has a path loss with respect to the
first two users equal to 12 dB. Fig. 4 corresponds to the
application of SA, whereas Fig. 5 corresponds to the
Lagrangian gradient-based algorithm with a μ parameter
equal to 0.001. We conclude that with the same
computational load, the SA can fulfill the constraints,
whereas the gradient based technique decreases
importantly the speed of convergence, as the solution is
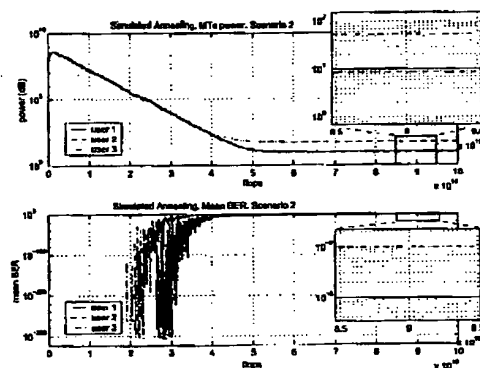nearer from them.

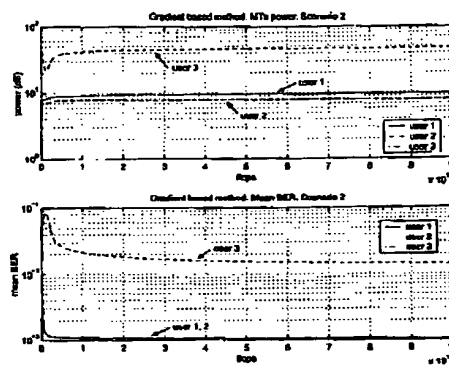Fig. 4. Performance of the Simulated Annealing
algorithm in scenario 2.

Fig. 5. Performance of the gradient based algorithm in
scenario 2.

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